A Probabilistic Constraint Approach for Robust Transmit Beamforming with Imperfect Channel Information

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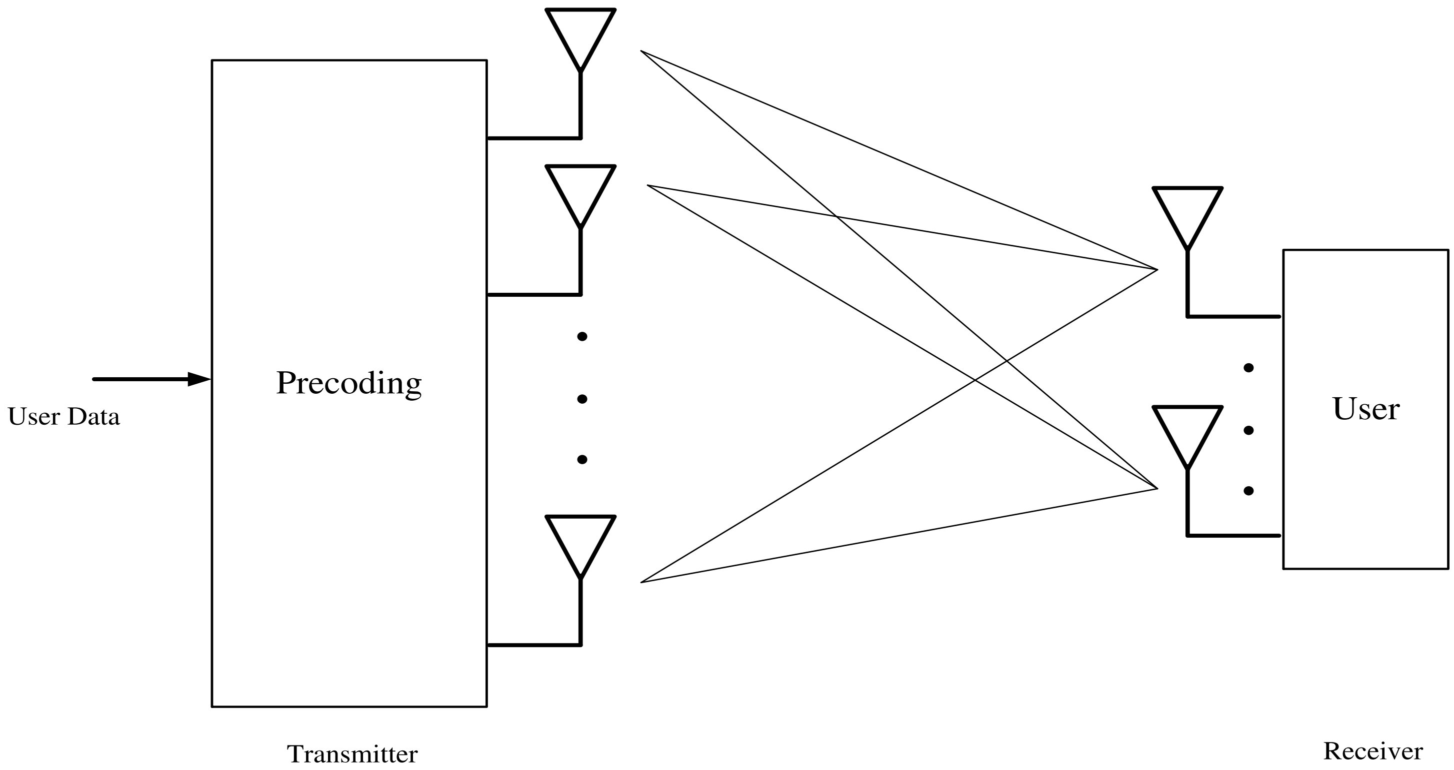
Motivation

- Transmit beamforming is a powerful technique for enhancing performance of wireless communication systems. Conventional approaches assume perfect channel state information at the transmitter, which is typically not available in practice.
- Existing robust designs focus on average or worst case performance. The former has severe consequences in the large error case, while the latter leads to an overall conservative performance.
- The probabilistic constraint approach maximizes the average signal-to-noise ratio (SNR) and takes the extreme conditions into account proportionally.

Multi-Input Multi-Output (MIMO) System



System Model

- Consider a single user system with N_t transmit antennas and N_r receive antennas.
- The signal $\mathbf{s}$ is spread over the precoding matrix $\mathbf{C}$ and transmitted through a flat fading channel $\mathbf{H}$.
- The receive signal is given by

$$\mathbf{y} = \mathbf{H}\mathbf{C}\mathbf{s} + \mathbf{w}$$

where $\mathbf{w}$ represents an additive white Gaussian noise.

Design Criterion: SNR

- Signal-to-Noise Ratio (SNR) is selected as design criterion because of its
  - mathematical simplicity,
  - close relation to SER, BER and MMSE at receiver.

- Assuming *perfect* channel information at the receiver, the SNR from maximum ratio combining (MRC) is given by

$$\mathrm{SNR} = \frac{E_s}{N_0} \mathrm{tr}\{\mathbf{C}^H \mathbf{H}^H \mathbf{H} \mathbf{C}\},$$

where E_s: signal energy, N_0: noise power.

Imperfect Channel State Information (CSI)

- In the presence of prefect CSI, maximizing SNR leads to conventional one directional beamforming which allocates all power on the strongest eigenmode of $\mathbf{H}$.
- In practice, only channel estimate $\hat{\mathbf{H}}$ is available,

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E}.$$

The error matrix $\mathbf{E}$ consists of i.i.d. complex normally distributed entries.

- Standard transmit beamforming degrades significantly!

Existing Robust Designs

- *Conventional Stochastic Approach*
  - use channel statistics (mean or covariance),
  - focus on average system performance,
  - pay no attention to extreme errors.
- *Maximin Approach*
  - consider deterministic errors,
  - optimize worst-case performance,
  - overall conservative performance.

Probabilistic Constraint Approach

- The probabilistic constraint approach is more flexible than the stochastic and worst case approaches.
- It maximizes overall performance while providing quality control at worst case.
- Challenges:
  - probabilistic constraint $\Rightarrow$ deterministic one
  - computational efficiency

Probabilistic Constrained Optimization I

Our design leads to the following stochastic optimization problem:

$$\max_{\mathbf{C}} \mathbb{E}(SNR)$$

subject to

$$\Pr\{SNR \leq \gamma_{th}\} \leq p_{out}$$

Power Constraint

* The received Signal-to-Noise Ratio at the receiver, SNR, is a random variable due to channel estimation errors.

Objective Function

- Assuming $\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E}$, SNR is given by

$$f(\hat{\mathbf{H}}, \mathbf{E}) = \frac{E_s}{N_0} \mathrm{tr}\{\mathbf{C}^H (\hat{\mathbf{H}} + \mathbf{E})^H (\hat{\mathbf{H}} + \mathbf{E}) \mathbf{C}\}.$$

- The objective function is the average SNR (with respect to $\mathbf{E}$)

$$\mathbb{E}\left[f(\hat{\mathbf{H}}, \mathbf{E})\right] = \frac{E_s}{N_0} \mathrm{tr}\{\mathbf{U}_c \mathbf{D}_c \mathbf{U}_c^H (\mathbf{U}_h \mathbf{D}_h \mathbf{U}_h^H + \sigma_e^2 N_r \mathbf{I}_{N_t})\},$$

where

$$\mathbf{C}\mathbf{C}^H = \mathbf{U}_c \mathbf{D}_c \mathbf{U}_c^H, \; \hat{\mathbf{H}}^H \hat{\mathbf{H}} = \mathbf{U}_h \mathbf{D}_h \mathbf{U}_h^H.$$

Objective Function (Cont'd)

- The average SNR can be maximized separately over the unitary matrix $\mathbf{U}_c$ and the diagonal matrix $\mathbf{D}_c = \mathrm{diag}(d_1, d_2, \ldots, d_{N_t})$.
- Inserting the optimal solution for $\mathbf{U}_c$ with $\mathbf{U}_c^H \mathbf{U}_h = \mathbf{I}$, we obtain

$$\bar{f}(\mathbf{D}_c) = \frac{E_s}{N_0} \mathrm{tr}\{\mathbf{D}_c(\mathbf{D}_h + \sigma_e^2 N_r \mathbf{I}_{N_t})\}.$$

- $\bar{f}(\mathbf{D}_c)$ depends only on $\mathbf{D}_c$. The design becomes a power allocation problem.

Probabilistic Constraint

- In the presence of large errors, the system performance is controlled by keeping the probability that SNR becomes smaller than a pre-specified level γ_{th} low.
- Mathematically

$$\Pr\{f(\hat{\mathbf{H}}, \mathbf{E}) \leq \gamma_{th}\} \leq p_{\rm out}, \quad (1)$$

where $\Pr\{A\}$ denotes the probability of the event A.

Reformulation of Probabilistic Constraint

Proposition The probabilistic constraint (1) can be replaced by the following convex constraint

$$\prod_{i=1}^{N_t} \left(\frac{1}{d}i \left[\frac{\bar{\gamma}/2}{1+\delta_i/n_i} \right] \right)^{n_i/2} \leq p_{\text{out}}. \qquad (2)$$

where δ_i represents the noncentrality parameter of $\chi^2_{n_i}(\delta_i)$-distribution and $n_i = 2N_r$. If (2) holds, then (1) holds.

* With (2), the original stochastic optimization problem is transformed into a convex optimization problem.

Probabilistic Constrained Optimization II

Having derived the compact expressions, our design can be formulated as

$$\max_{\mathbf{D}_c} \mathrm{tr}\{\mathbf{D}_c(\mathbf{D}_h + \sigma_e^2 N_r \mathbf{I}_{N_t})\},$$

$$\text{subject to} \quad \prod_{i=1}^{N} \left(\frac{1}{d_i}\left[\frac{\bar{\gamma}}{2(1+\delta_i/n_i)}\right]\right)^{n_i/2} \leq p_{\text{out}},$$

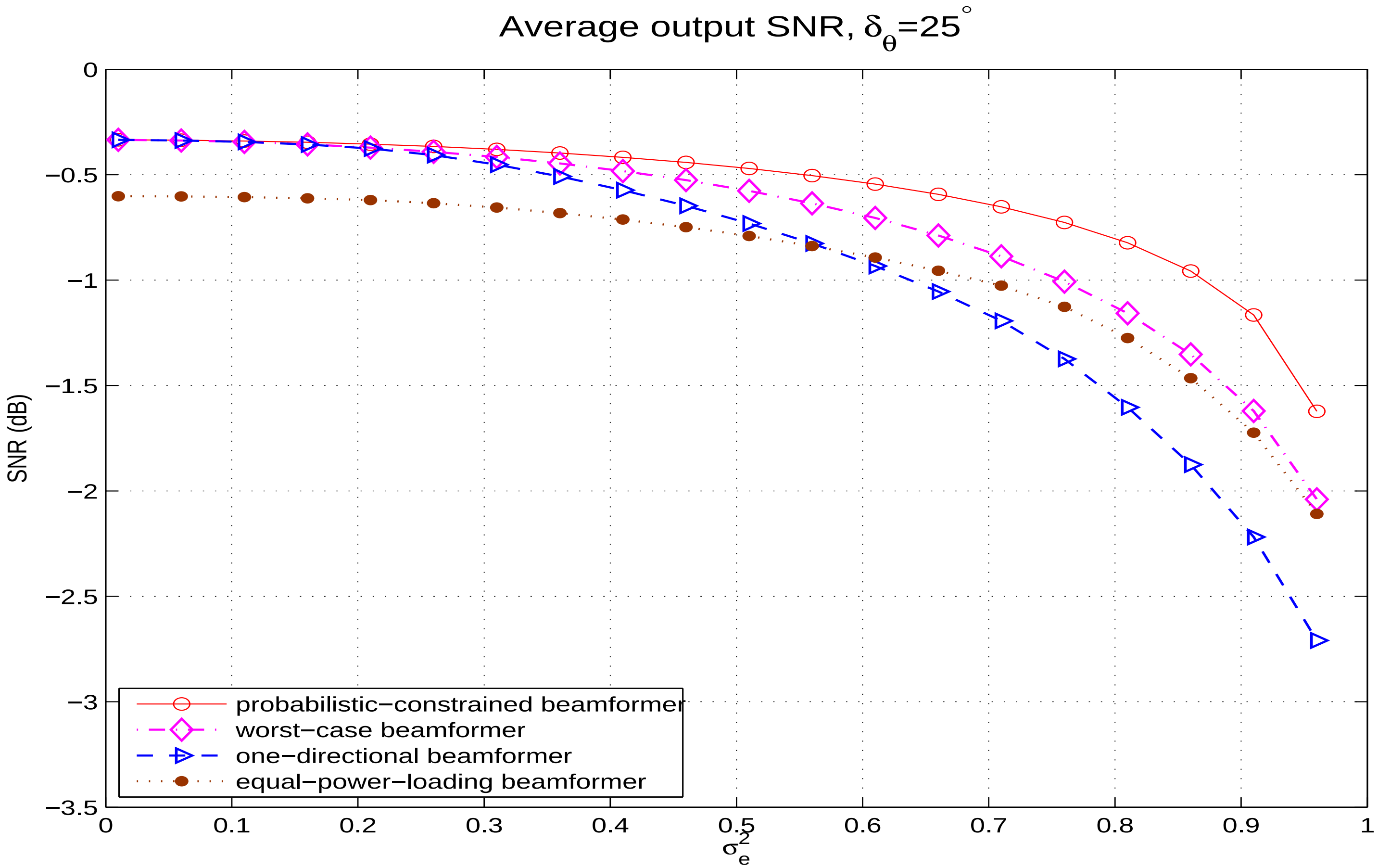
$$\mathrm{tr}\{\mathbf{D}_c\} \leq 1, \qquad (3)$$

$$d_i \geq 0, \quad i = 1, \cdots, N_t,$$

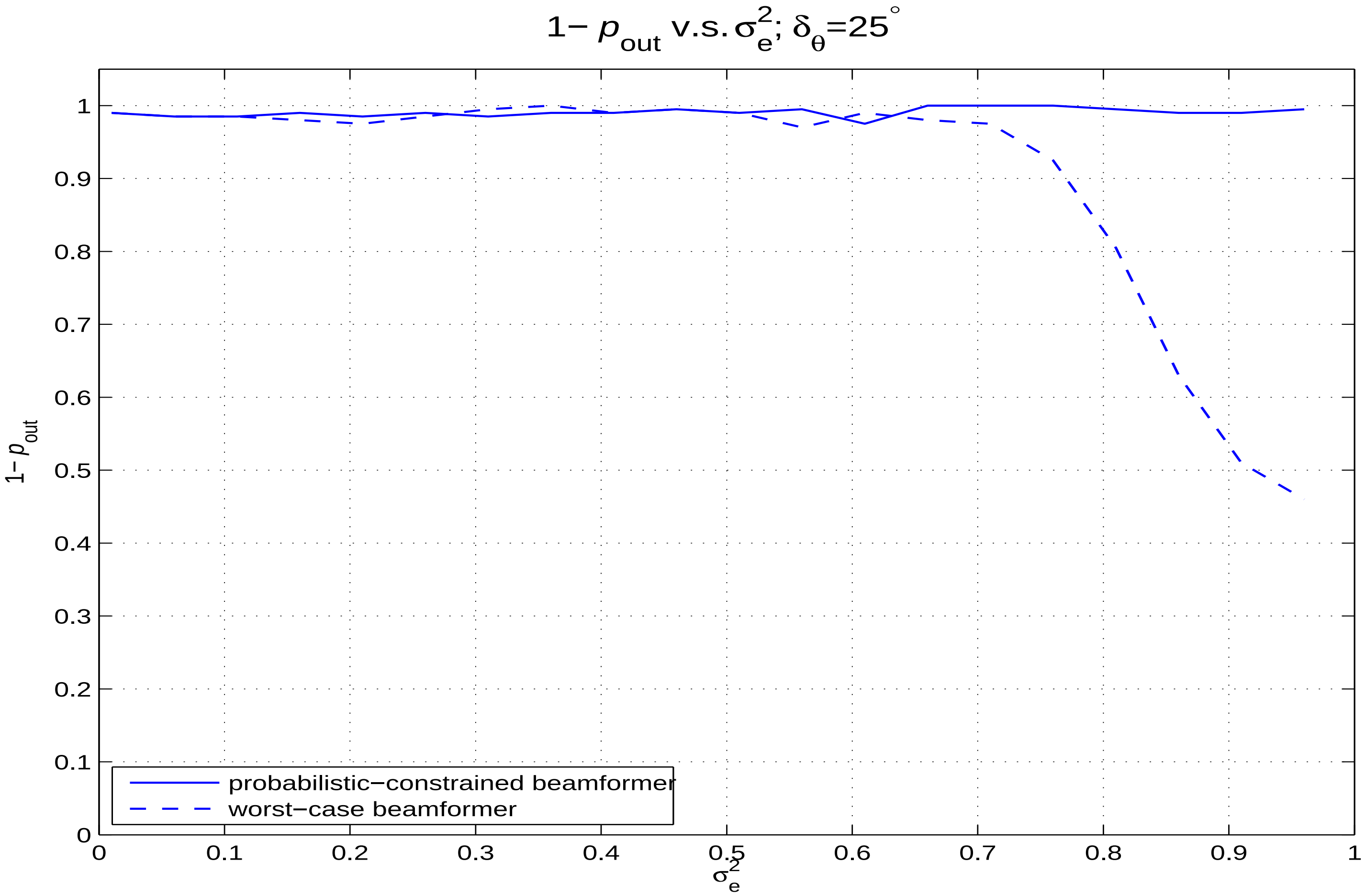
where $\bar{\gamma} = \gamma_{th}(\frac{E_s}{N_0}\sigma_e^2)^{-1}$ and (3) is the power constraint.

Simulation

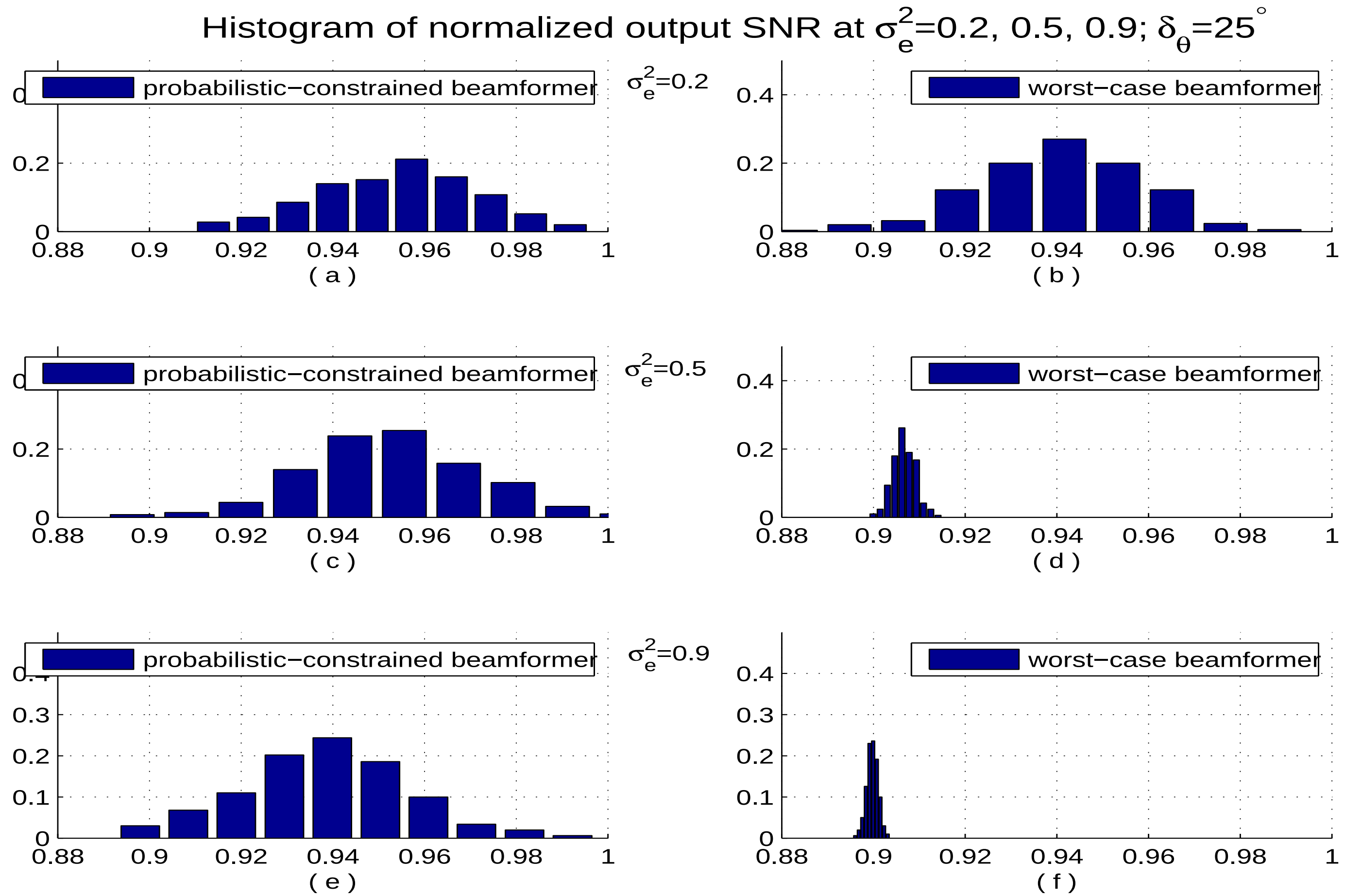
- We consider a single-user MIMO system with $N_t = 4$ transmit antennas and $N_r = 3$ receive antennas.
- We also compare the proposed approach with 1. the conventional one-directional beamformer, 2. two-directional, 3. equal-power loading beamformer and 4. the worst case approach.
- The outage probability p_{out} is 10% and the normalized SNR threshold $\bar{\gamma}$ is 0.9. The error variance $0 \leq \sigma_e^2 \leq 1$.
- Correlated fading with fixed antenna spacing $d = 0.5\lambda$ and angle spread $\delta_\theta = 25^\circ$.



Average SNR vs. error variance σ_e^2. Correlated fading with angular spread $\delta_\theta = 25^\circ$.



$(1 - p_{out})$ vs error variance σ_e^2. Correlated fading with angular spread $\delta_\theta = 25^\circ$.



Histogram of normalized SNR for $\sigma_e^2 = 0.2, 0.5, 0.9$. Correlated fading with angular spread $\delta_\theta = 25^\circ$.

Conclusions

- We applied the probabilistic constraint approach for transmit beamforming design in general MIMO systems.
- The proposed method maximizes the average SNR performance and guarantees robustness against channel estimation errors.
- The probabilistic constraint was transformed to a convex one. Computational complexity is the same as most robust designs.
- The proposed beamformer achieves the highest robustness and best system performance among existing robust designs.

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