Robust and Distributed Beamforming

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Outline

The Wireless Challenges

- 4G Goals:
  - Applications: Internet, data, voice, videoconference
  - Anytime-anywhere
  - Technical Expectations
    - 100+ Mbps for fast moving users and 1+ Gbps for slow moving users
    - More simultaneous users per cell
    - Global roaming across multiple networks
- Problems:
  - Channel fading
  - Frequency spectrum scarcity
  - Power constraints
  - Cost consideration, e.g., complexity

Emerging Solutions: MIMO & Relay

- MIMO antenna systems
  - Increase the spectral efficiency (**multiplexing gain**)
  - Improve the link reliability (**diversity gain**)
- Included in IEEE WLAN 802.11n, WiMAX 802.16-2004, 802.16e, 3GPP Releases 7 and 8 (LTE), etc.
- The use of MIMO is however constrained by space and costs
- Relay is a promising means to extend the range of communication by forwarding message to the destination terminal
- Included in IEEE 802.11s-WLAN, IEEE 802.16j-WMAN, and IEEE 802.20-MBWA, etc.

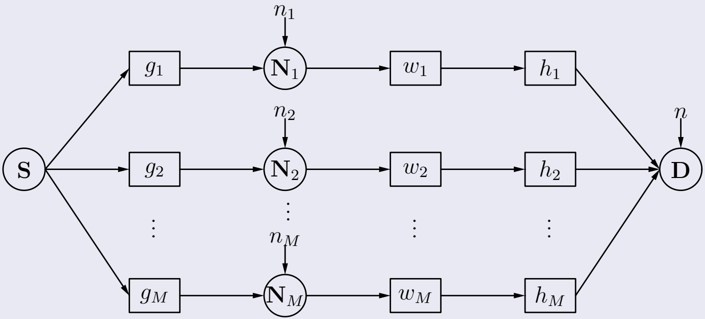
- User cooperation rekindles the idea of relay, i.e., a user can act as a relay to provide a **diversity path** for another user
- Collaborative use of relays, if optimized properly, could form a **virtual MIMO** antenna system

Our Aim

- **This talk** is about the joint optimization of relays for forming a virtual signal beam for enhancing the signal-to-noise ratio (SNR) at the destination terminal with the aid of imperfect channel state information (CSI) at the original sender and the relay terminals
- We coin the approach "Collaborative-Relay Beamforming (CRBF)"

Single Source, Single Destination and Multiple Relays

- Broken link between the source and destination terminals
- Relays work in "Amplify-and-Forward" manner synchronously



Previous Works

- Most researches out there deal with performance analysis of fixed-gain relay channels (e.g., Laneman *et al.* T-IT 2004, Scaglione *et al.* SP Mag. 2006, Sendonaris *et al.* T-COM 2003, etc.)
- Limited studies on variable-gain relays:
  - Perfect CSI–The optimal power control and a distributive scheme with a sum-relay-power constraint were addressed (Larsson *et al.* 2003)
  - Partial CSI–Relays are optimised based on an approximated mean SNR for a given sum-relay-power (Yi *et al.* JSAC 2007)
  - Imperfect CSI–Robust power allocation over relays was first studied in the presence of ellipsoidal uncertainty sets (Quek *et al.* JSTSP 2007) where a suboptimal solution was proposed based on bounding

More Factors to Consider

1) CSI errors, 2) Individual constraints, 3) Distributed implementation ...

Channel Uncertainty Model: A Hypersphere

- The CSI available at $\mathbf{S}$, denoted as $\hat{\mathbf{h}}$, is modeled as

$$\hat{\mathbf{h}} = \mathbf{h} + \Delta\mathbf{h},$$

where $\mathbf{h}$ is the actual CSI vector and $\Delta\mathbf{h}$ denotes the CSI error vector which is assumed to be bounded in the form by $\Delta\mathbf{h} \in \mathcal{U}$ such that

$$\mathcal{U} = \left\{ \mathbf{a} \in \mathbb{C}^M : \mathbf{a}^\dagger \mathbf{a} \leq \sum_{m=1}^{M} \rho_m^2 \right\}$$

- Such model is particularly useful to model the estimation and quantization error in CSI (Shenouda *et al.* JSTSP 2007)
- Probabilistic models exist if CSI estimation errors are to be considered (Pascual-Iserte *et al.* T-SP 2006, Wiesel *et al.* T-WC 2007)

The **received SNR** is given by

$$\Gamma(\mathbf{w}, \Delta\mathbf{h}) = \frac{P_s \left| \sum_{m=1}^{M} (\hat{h}_m + \Delta h_m) g_m l_m w_m \right|^2}{\sum_{m=1}^{M} |\hat{h}_m + \Delta h_m|^2 |l_m|^2 |w_m|^2 N_m + N_0},$$

where $l_m = \dfrac{1}{\sqrt{|g_m|^2 P_s + N_m}}$

The Two Closely-Related Problems for Optimization

$\mathcal{Q}_1$: To maximize the worst SNR with relay-power constraints

$$\max_{\substack{|w_m|^2 \le p_m \forall m \\ \gamma \ge 0}} \gamma \quad \text{s.t.} \quad \min_{\triangle\mathbf{h} \in \mathcal{U}} \Gamma(\mathbf{w}, \Delta\mathbf{h}) \ge \gamma$$

$\mathcal{Q}_2$: To minimize the sum-power with SNR and power constraints

$$\min_{|w_m|^2 \le p_m \forall m} \|\mathbf{w}\|^2 \quad \text{s.t.} \quad \min_{\triangle\mathbf{h} \in \mathcal{U}} \Gamma(\mathbf{w}, \Delta\mathbf{h}) \ge \gamma$$

What Makes the Problem Intractable?

The constraint on the worst-case SNR can be rewritten as

$$\min_{\Delta\mathbf{h}\in\mathcal{U}} f(\Delta\mathbf{h}) \geq 0, \text{ where}$$

$$f(\Delta\mathbf{h}) \triangleq P_s \left| \sum_{m=1}^{M} (\hat{h}_m + \Delta h_m) g_m l_m w_m \right| - \sqrt{\gamma \left(\sum_{m=1}^{M} |\hat{h}_m + \Delta h_m|^2 |l_m|^2 |w_m|^2 N_m + N_0 \right)}$$

Mathematical Difficulties

$f(\Delta\mathbf{h})$ is non-convex in $\Delta\mathbf{h}$ and it is known in (Ben-Tal *et al.* 2007) that this constraint is intractable with a hypersphere uncertainty set $\mathcal{U}$

Existing Approach: Lower Bounding (Quek et al. JSTSP 2007)

Replace $\min_{\Delta\mathbf{h}\in\mathcal{U}} f(\Delta\mathbf{h})$ by its lower bound

$$\underline{f}(\mathbf{w}) \triangleq P_s \left| \sum_{m=1}^{M} \hat{h}_m g_m l_m w_m \right| - P_s \sum_{m=1}^{M} \rho_m \left| g_m l_m w_m \right| - \sqrt{\gamma \left(\sum_{m=1}^{M} (|\hat{h}_m| + \rho_m)^2 |l_m|^2 |w_m|^2 N_m + N_0 \right)} \leq f(\mathbf{w}, \Delta\mathbf{h})$$

The Lower Bounding Approach

- The constraint becomes $\underline{f}(\mathbf{w}) \geq 0$
- The constraint is convex and easily handled
- The bound is generally VERY LOOSE and the SNR performance is severely compromised!

With $\mathcal{U}$, the worst-case SNR constraint can be written as

$$(\tilde{\mathbf{h}}+\Delta\tilde{\mathbf{h}})^{\dagger}\mathbf{Q}(\tilde{\mathbf{h}}+\Delta\tilde{\mathbf{h}}) \geq 0 \ \forall \triangle \mathbf{h} \in \mathcal{U} \ \text{ where } \ \tilde{h}_m \triangleq |\hat{h}_m|,$$

$$v_m \triangleq \frac{g_m l_m w_m |\hat{h}_m|}{\hat{h}_m^*} \ \text{ and } \ \mathbf{Q} \triangleq P_s \mathbf{v}\mathbf{v}^{\dagger} - \gamma \begin{bmatrix} \frac{|v_1|^2 l_1^2 N_1^2}{|g_1|^2} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{|v_M|^2 l_M^2 N_M^2}{|g_M|^2} \end{bmatrix}$$

S-Procedure (Fradkov *et al.* 1973)

Using S-Procedure, the worst-case SNR constraint is equivalent to

$$\begin{bmatrix} \tilde{\mathbf{h}}^T\mathbf{Q}\tilde{\mathbf{h}} - \gamma N_0 - s\sum_{m=1}^{M}\rho_m^2 & \tilde{\mathbf{h}}^T\mathbf{Q} \\ \mathbf{Q}\tilde{\mathbf{h}} & \mathbf{Q}+s\mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \ \exists s \geq 0$$

Rank Relaxation Leads to an SDP

Introducing the matrix variable $\mathbf{V} = \mathbf{v}\mathbf{v}^\dagger$ and removing the rank constraint of $\mathbf{V}$, we can then reexpress $\mathcal{Q}_2$ as the following (convex) semi-definite programming (SDP) problem

$$\tilde{\mathcal{Q}}_2 : \begin{cases} \min\limits_{s \geq 0, \mathbf{V} \succeq \mathbf{0}} \ \text{trace}(\mathbf{V}\mathbf{G}) \\ \quad \text{s.t.} \ \begin{bmatrix} \tilde{\mathbf{h}}^T \mathbf{Q} \tilde{\mathbf{h}} - \gamma N_0 - s \sum_{m=1}^{M} \rho_m^2 & \tilde{\mathbf{h}}^T \mathbf{Q} \\ \mathbf{Q} \tilde{\mathbf{h}} & \mathbf{Q} + s\mathbf{I} \end{bmatrix} \succeq \mathbf{0} \\ \quad [\mathbf{V}]_{m,m} \leq p_m |g_m|^2 l_m^2 \ \forall m \\ \quad [\mathbf{G}]_{m,m} \triangleq \dfrac{1}{|g_m|^2 l_m^2} \ \forall m \end{cases}$$

The Rank Issue

Theorem 1

If $\tilde{h}_m > \sqrt{\sum_{m=1}^{M} \rho_m^2}\ \forall m$, then a rank-1 optimal solution to $\tilde{\mathcal{Q}}_2$ is always guaranteed and hence the solution to the SDP gives the exact robust optimal solution for CRBF

How Well It Works?

Table: Probability of Guaranteed Optimality (Rank-1 Solution)

p/N_0 (dB)	0	5	10	15	20
$M = 4, \rho = 0.01$	1	1	1	1	1
$M = 4, \rho = 0.1$	1	1	1	0.98	0.93
$M = 10, \rho = 0.01$	1	1	1	1	1
$M = 10, \rho = 0.1$	1	1	1	0.98	0.91

Distributed Implementation

Some Issues Remain

- For centralized implementation, the source terminal needs to inform each individual relay its own AF weight, w_m, which takes extra bandwidth to do so and can be costly, if the number of cooperative relay terminals is large
- For the solution to be attractive and viable, it has to be more scalable in implementation (A distributed implementation is NEEDED!)

Distributed implementation is possible after examining the structure of the optimal solution of the relaxed convex SDP problem.

How It Works?

- The source terminal first solves the problem using the proposed algorithm
- The source terminal broadcasts several global optimizer variables (4 positive numbers) to all relaying terminals
- Each individual relaying terminal derives its beamforming weight based on the received common information and its local estimated CSI

Distributed implementation – Optimal Structure

Theorem 2

If the resulting solution to $\tilde{Q}_2$ satisfies $s\mathbf{I} + \mathbf{Q} \succ \mathbf{0}$, then the optimal beamforming weight v_m is given by

$$(v_m)_{\text{opt}} = \frac{\tau\left(\sqrt{\alpha}\tilde{h}_m + \frac{u_m}{\sqrt{\alpha}}\right)}{[\mathbf{G}]_{m,m} + \beta_m + \frac{\gamma N_m}{|g_m|^2}\left(\sqrt{\alpha}\tilde{h}_m + \frac{u_m}{\sqrt{\alpha}}\right)^2},$$

where α, β_m, u_m are dual variables and τ is chosen to satisfy the relay's individual power constraint

Distributed Implementation, u_m

Our Approach

- Define $x_m \triangleq \frac{u_m}{\alpha}$, which is actually the worst CSI error and the solution to the following problem

$$\min_{\|\mathbf{x}\|^2 \leq M\rho^2} \sum_{m=1}^{M} \frac{(\tilde{h}_m + x_m)^2}{\frac{1}{|g_m|^2 l_m^2} + \beta_m + \frac{\gamma \alpha N_m}{|g_m|^2}(\tilde{h}_m + x_m)^2}$$

- It should satisfy the optimality condition

$$\frac{\left(\frac{1}{|g_m|^2 l_m^2} + \beta_m\right)(\tilde{h}_m + x_m)}{\left[\frac{1}{|g_m|^2 l_m^2} + \beta_m + \frac{\gamma \alpha N_m}{|g_m|^2}(\tilde{h}_m + x_m)^2\right]^2} + \lambda x_m = 0$$

where λ is a solution to the constraint $\|\mathbf{x}\|^2 = M\rho^2$ and can be known from the source terminal's information

Distributed Implementation, β_m, v_m

Our Approach (cont')

- β_m must satisfy

$$\beta_m(v_m - \sqrt{p_m|g_m|^2 l_m^2}) = 0 \ \forall m$$

- This means that if $(\beta_m)_{\text{opt}} \neq 0$, the optimal weight is simply $(v_m)_{\text{opt}} = \sqrt{p_m|g_m|^2 l_m^2}$. Otherwise, $(\beta_m)_{\text{opt}} = 0$

The Optimal v_m

- The final solution is therefore given by

$$(v_m)_{\text{opt}} = \min\left\{ \sqrt{p_m|g_m|^2 l_m^2}, \frac{\tau\left(\tilde{h}_m + x_m\right)}{\frac{1}{|g_m|^2 l_m^2} + \frac{\gamma\alpha N_m}{|g_m|^2}(\tilde{h}_m + x_m)^2} \right\}$$

- The common parameters are λ, $M\rho^2$, τ and $\gamma\alpha$

Setup and Benchmarks

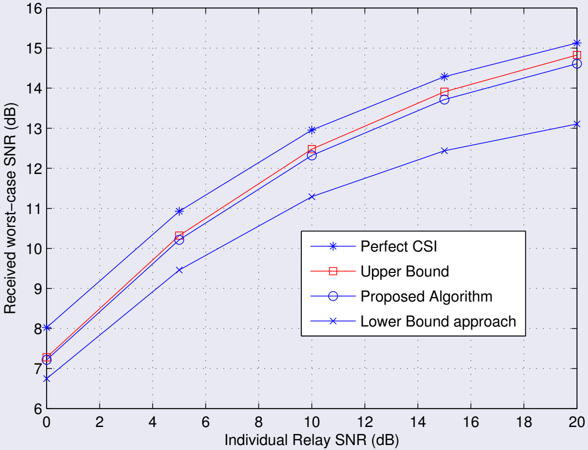
- Setup
  - $\mathbf{g} \sim \mathcal{CN}(0, \sigma_g^2\mathbf{I})$ and $\mathbf{h} \sim \mathcal{CN}(0, \sigma_h^2\mathbf{I})$ and $\sigma_g^2 = \sigma_h^2 = 1$
  - Assume $\rho_m = \rho$, and $p_m = p \; \forall m$
  - W.L.O.G., set $N_m = N_0 = 1 \; \forall m$
- Benchmarks
  - Optimal beamforming design with perfect CSI
  - The worst-case SNR upper bound, with a shrinking Hypercube uncertainty region

$$\mathcal{U}_1 = \left\{ \mathbf{a} \in \mathbb{R}^M : |a_m| \leq \rho_m \; \forall m \right\}$$

  - Lower bounding approach

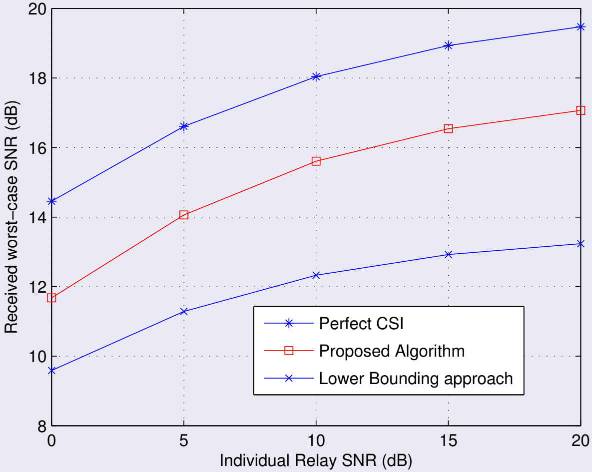
Results (1)

Parameters: $M = 4$, $\rho^2 = 0.01$, P_s/N_0=10 (dB)



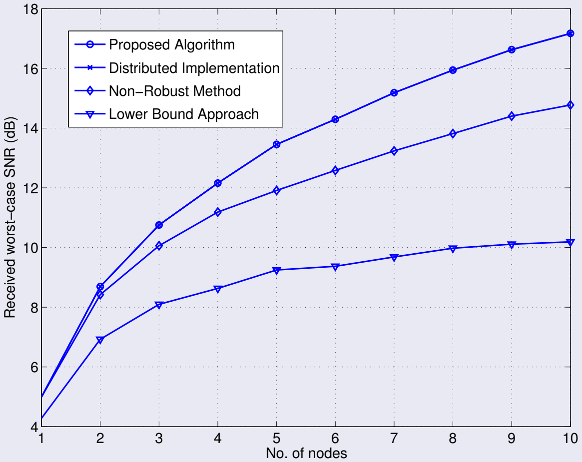
Results (2)

Parameters: $M = 10$, $\rho^2 = 0.1$, P_s/N_0=10 (dB)



Results (3)

Parameters: $\rho^2 = 0.01$, P_s/N_0=10 (dB)



Conclusion

- We have investigated the worst-case destination SNR maximization problem of using CRBF with individual relay peak-power constraints
- With CSI uncertainty being modeled as ellipsoids, it is possible to obtain the optimal solution using S-Procedure and rank relaxation and a distributed implementation algorithm is also possible
- Substantial performance gain over existing methods has been observed

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- With CSI uncertainty being modeled as ellipsoids, it is possible to obtain the optimal solution using S-Procedure and rank relaxation and a distributed implementation algorithm is also possible
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Related Publications

1. G. Zheng, K. K. Wong, A. Paulraj, and B. Ottersten, Robust collaborative-relay beamforming, IEEE Transactions on Signal Processing, Vol. 57, No. 8, pp. 3130–3143, August 2009.
2. G. Zheng, K. K. Wong, A. Paulraj, and B. Ottersten, Collaborative-relay beamforming with perfect CSI: Optimum and distributed implementation, IEEE Signal Processing Letters, Vol. 16, No. 4, pp. 257–260, April 2009.