A Probabilistic Approach for Robust Leakage-based MU-MIMO Downlink Beamforming with Imperfect Channel State Information

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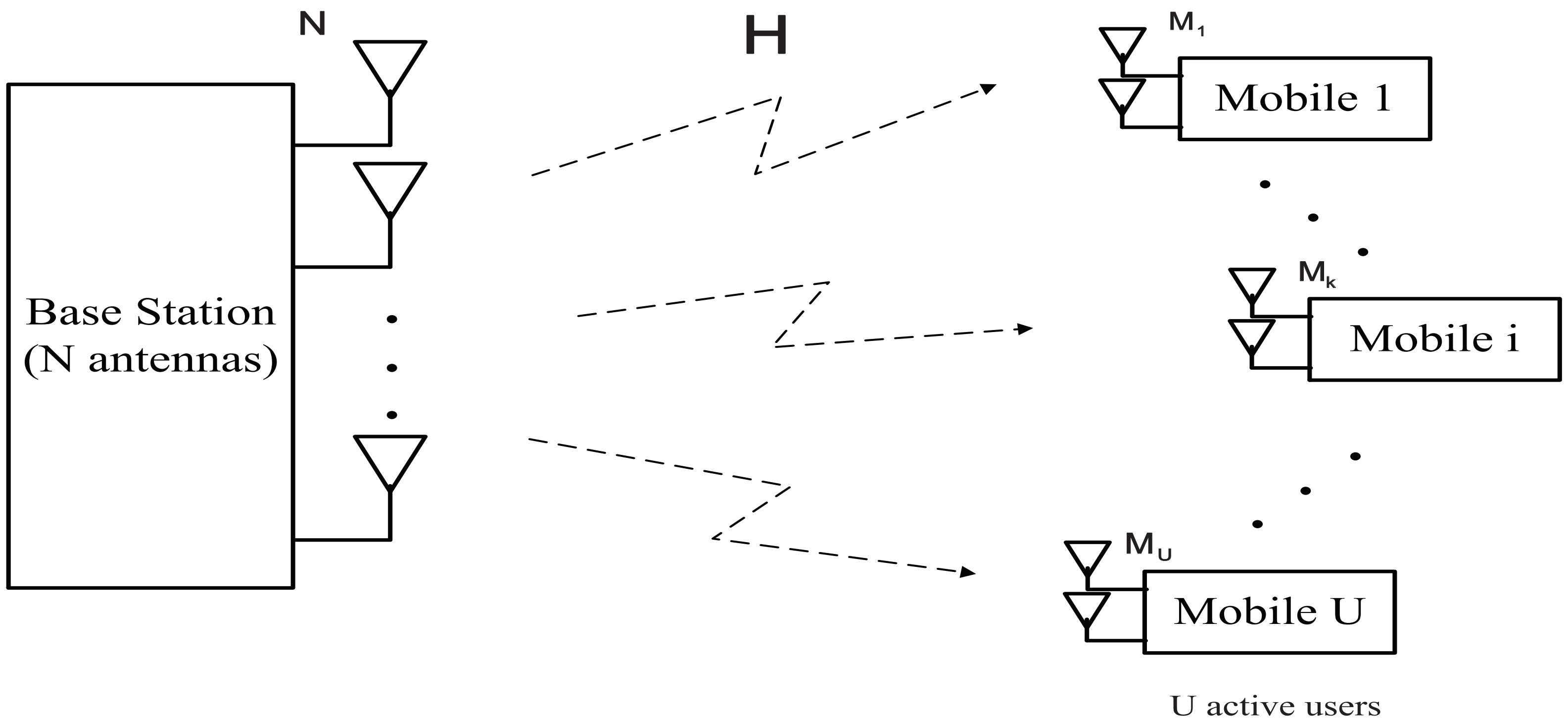
Motivation

- Transmit beamforming in Multi User MIMO (MU-MIMO) systems is a powerful signal processing technique for increasing system capacity. Conventional approaches assume perfect channel state information at the transmitter, which is typically not available in practice.
- Existing robust designs focus on average or worst case performance. The former has severe consequences in the presence of large channel errors, while the latter leads to an overall conservative performance.
- The probabilistic constraint approach maximizes the average system performance and takes extreme conditions proportionally. It has the best performance among existing approaches.

MU-MIMO System



System Model

- A downlink MU-MIMO system consists of one base station and K users.
  The base station employs N transmit antennas.
  The ith user is equipped with M_i antennas.
- For the ith user, the receive signal is given by

  $$\mathbf{y}_i = \mathbf{H}_i\, \mathbf{c}_i\, s_i + \sum_{k=1,k\neq i}^{K} \mathbf{H}_i\, \mathbf{c}_k\, s_k + \mathbf{n}_i,$$

  s_i: transmitted signal, $\mathbf{c}_i$: bemaforming vector
  $\mathbf{H}_i$: channel matrix, $\mathbf{n}_i$: additive Gaussian noise.

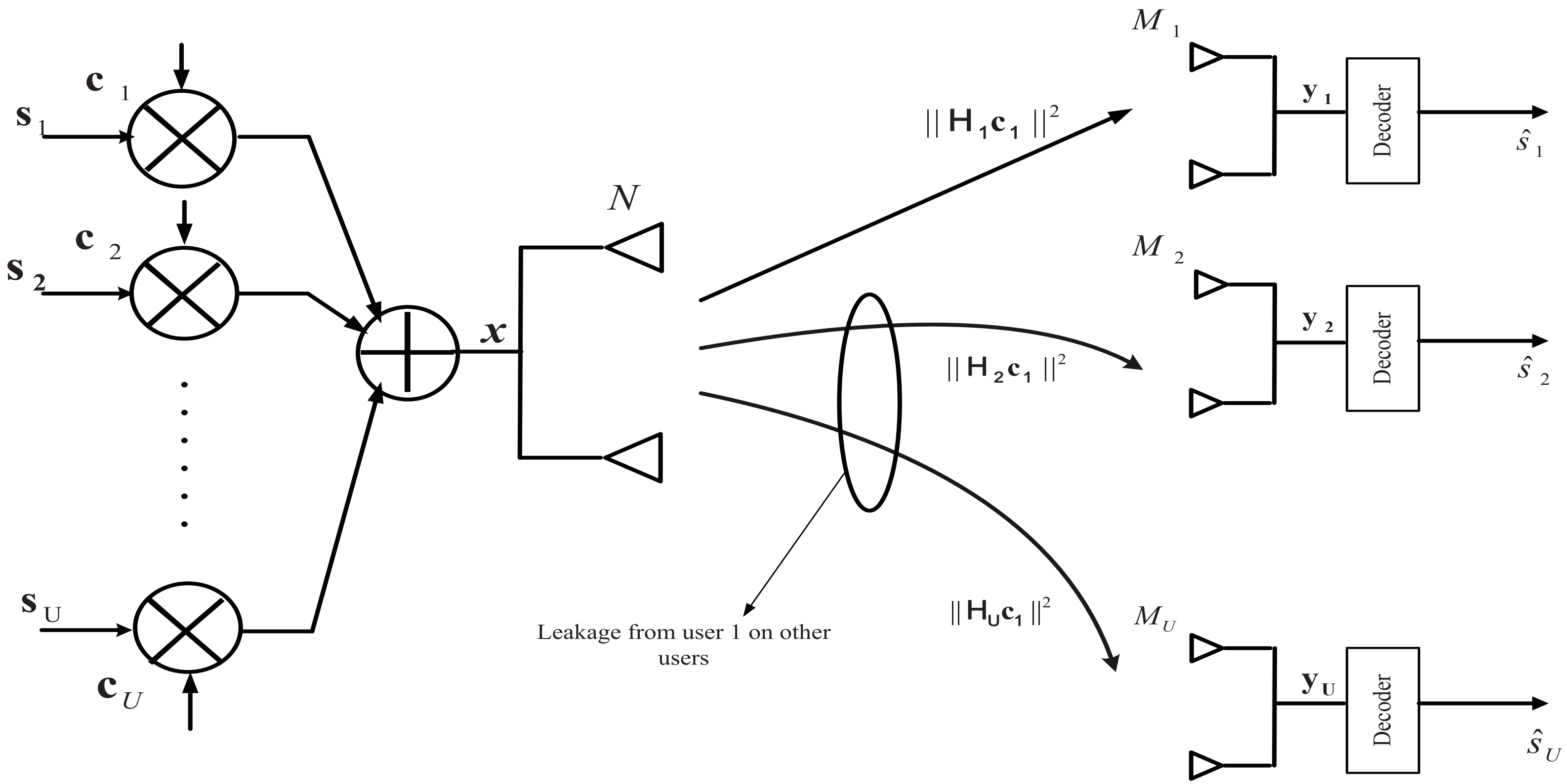
Transmit Beamforming in MU-MIMO

- Main challenge is to completely suppress co-channel interference (CCI) from other users, which requires perfect channel information.

- Existing transmit beamforming designs employ various performance measures including Minimum Mean Squared Errors (MMSE), Signal to Interference and Noise Ratio (SINR), etc.

- We adopt the Signal-to-Leakage and Noise Ratio (SLNR) criterion because it is mathematically tractable and admits closed form solution in the prefect channel case.

Leakage Power



Signal-to-Leakage-Noise Ratio: Perfect CSI

- The SLNR for the ith user in the perfect channel case is given by

$$\mathrm{SLNR}_i = \frac{||\mathbf{H}_i \mathbf{c}_i||_F^2}{M_i \sigma_i^2 + \sum_{k=1, k \neq i}^{K} ||\mathbf{H}_k \mathbf{c}_i||_F^2} = \frac{||\mathbf{H}_i \mathbf{c}_i||_F^2}{M_i \sigma_i^2 + ||\tilde{\mathbf{H}}_i \mathbf{c}_i||_F^2} ,$$

where $\tilde{\mathbf{H}}_i = [\mathbf{H}_1^T, \ldots, \mathbf{H}_{i-1}^T, \mathbf{H}_{i+1}^T, \ldots, \mathbf{H}_K^T]^T$,
$||\mathbf{H}_i \mathbf{c}_i||_F^2$: signal power, $||\tilde{\mathbf{H}}_i \mathbf{c}_i||_F^2$: leakage power

- Maximizing SLNR_i leads to a closed form solution for $\mathbf{c}_i$.
- The nonrobust design degrades dramatically in the presence of channel errors.

SLNR: Imperfect CSI

- The channel estimate$\mathbf{H}_{i_p}$ is related to true model $\mathbf{H}_i$ as follows

$$\mathbf{H}_i = \mathbf{H}_{i_p} + \mathbf{E}_i \ ,$$

$\mathbf{E}_i$: error matrix with i.i.d. normally distributed entries.

- Signal-to-Leakage Ratio is given by

$$\mathrm{SLR}_i = \frac{\mathbf{c}_i^H(\mathbf{H}_{i_p}+\mathbf{E}_i)^H(\mathbf{H}_{i_p}+\mathbf{E}_i)\mathbf{c}_i}{M_i\sigma_i^2+\mathbf{c}_i^H(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}}_i)^H(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}}_i)\mathbf{c}_i} \ .$$

$$\tilde{\mathbf{H}}_{i_p} = [\mathbf{H}_{1_p}^T, \cdots, \mathbf{H}_{(i-1)_p}^T, \mathbf{H}_{(i+1)_p}^T, \cdots, \mathbf{H}_{K_p}^T]^T,$$

$$\tilde{\mathbf{E}}_i = [\mathbf{E}_{1_p}^T, \cdots, \mathbf{E}_{(i-1)_p}^T, \mathbf{E}_{(i+1)_p}^T, \cdots, \mathbf{E}_{K_p}^T]^T.$$

Existing Robust Designs

- *Conventional Stochastic Approach*
  - use channel statistics (mean or covariance),
  - focus on average system performance,
  - pay no attention to extreme errors.
- *Maximin Approach*
  - consider deterministic errors,
  - optimize worst-case performance,
  - overall conservative performance.

Probabilistic Constraint Approach

- The probabilistic constraint approach is more flexible than the stochastic and worst case approaches.
- It maximizes overall performance while providing quality control at worst case.
- Challenges:
  - probabilistic constraint $\Rightarrow$ deterministic one
  - computational efficiency

Robust Transmit Beamforming Based on SLNR

Robustness is achieved by maximizing average receive power of the ith user and keeping leakage power below a pre-specified level.

$$\begin{aligned}
\max_{\mathbf{c}_i} \quad & \mathbb{E}\left[\mathbf{c}_i^H(\mathbf{H}_{i_p}+\mathbf{E}_i)^H(\mathbf{H}_{i_p}+\mathbf{E}_i)\mathbf{c}_i\right] \ , \\
\text{s.t.} \quad & \Pr\left\{\mathbf{c}_i^H(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}}_i)^H(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}}_i)\mathbf{c}_i \geq \gamma_{th_i}\right\} \leq p_i \ , \\
& \mathrm{tr}\{\mathbf{c}_i\mathbf{c}_i^H\} \leq 1 \ , \\
& \mathrm{rank}(\mathbf{c}_i\mathbf{c}_i^H) = 1,
\end{aligned}$$

where γ_{th_i} is a pre-specified leakage power level, and p_i is a pre-specified probability.

Simplification

- To simplify notation, define $\mathbf{W}_i \triangleq \mathbf{c}_i \mathbf{c}_i^H$.
- *Rank relaxation*: Applying Lagrangian relaxation, the rank constraint $\mathrm{rank}(\mathbf{W}_i) = 1$ is replaced by $\mathrm{rank}(\mathbf{W}_i) \geq 0$.
- The *objective function* is given by

$$\mathrm{tr}\left\{\left(\mathbf{H}_{i_p}^H \mathbf{H}_{i_p} + \sigma_e^2 \mathbf{I}_N\right) \mathbf{W}_i\right\}.$$

- The *probabilistic constraint* is transformed to a convex, deterministic one by Markov inequality

$$\mathrm{tr}\left\{\left(\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} + n_i \sigma_e^2 \mathbf{I}_N\right) \mathbf{W}_i\right\} \leq p_i \gamma_{th_i} \ ,$$

where $n_i = \sum_{k=1,k\neq i}^{K} M_k$.

Reformulation

• The probabilistic constraint optimization is transformed to a convex optimization one as follows.

$$\max_{\mathbf{W}_i} \ \mathrm{tr}\left\{\left(\mathbf{H}_{i_p}^H \mathbf{H}_{i_p} + M_i \sigma_e^2 \mathbf{I}_N\right) \mathbf{W}_i\right\} \ ,$$

$$\text{s.t.} \ \ \mathrm{tr}\left\{\left(\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} + n_i \sigma_e^2 \mathbf{I}_N\right) \mathbf{W}_i\right\} \leq p_i \gamma_{th_i} \ ,$$

$$\mathrm{tr}\{\mathbf{W}_i\} \leq 1 \ , \qquad \mathbf{W}_i \geq 0, \qquad i = 1 \ , \ldots , K \ ,$$

where the matrix $\mathbf{W}_i$ is the design parameter.

• It can be shown that the proposed robust beamforming design places an upper bound for on average SLNR.

Multiple Stream MU-MIMO System

• The proposed approach is applied to multiple stream per user MU-MIMO.

• In addition to interferences from other users, the inter-stream-interferences also needs to be considered.

• We suggest a robust transmit beamforming design assisted by Alamouti code to mitigate the impact of multiple stream.

• Consider the simple case with data length $L_k = 2$. The transmit code block is given by

$$\mathbf{s}_k = \begin{bmatrix} s_{k,1} \\ s_{k,2} \end{bmatrix} \Rightarrow \mathbf{S}_k = \begin{bmatrix} s_{k,1} & -s_{k,2}^* \\ s_{k,2} & s_{k,1}^* \end{bmatrix},$$

Multiple Stream MU-MIMO System Model

- The transmitted signal matrix $\mathbf{X} \in \mathbb{C}^{N\times 2}$ can be presented as

$$\mathbf{X} = \sum_{k=1}^{K} \mathbf{C}_k \mathbf{S}_k \ ,$$

where $\mathbf{C}_k \in \mathbb{C}^{N\times 2}$ denotes the beamforming matrix.

- The data block received by user i can be written as

$$\mathbf{Y}_i = \mathbf{H}_i \sum_{i=1}^{K} \mathbf{C}_i \mathbf{S}_i + \mathbf{N}_i = \mathbf{H}_i \mathbf{C}_i \mathbf{S}_i + \mathbf{H}_i \sum_{k=1, k\neq i}^{K} \mathbf{C}_k \mathbf{S}_k + \mathbf{N}_i \ ,$$

where $\mathbf{N}_i \in \mathbb{C}^{M_i\times 2}$ is the noise matrix with entries being i.i.d complex normally distributed with zero mean and variance σ_i^2.

Robust Design in the Multiple Stream Case

The proposed beamforming design can be formulated as follows

$$\max_{\mathbf{C}_i} \quad \mathbb{E}\left[\mathrm{tr}\left\{\mathbf{C}_i^H(\mathbf{H}_{i_p}+\mathbf{E}_i)^H(\mathbf{H}_{i_p}+\mathbf{E}_i)\mathbf{C}_i\right\}\right] \ ,$$

$$\text{s.t.} \quad \Pr\left\{\mathrm{tr}\left\{\mathbf{C}_i^H(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}}_i)^H(\tilde{\mathbf{H}}_{i_p}+\tilde{\mathbf{E}}_i)\mathbf{C}_i\right\} \geq \gamma_{th_i}\right\} \leq p_i \ ,$$

$$\mathrm{tr}\{\mathbf{C}_i\mathbf{C}_i^H\} \leq 2 \ , \quad i = 1 \ , \ldots , K \ ,$$

where the optimization parameter is the precoding matrix $\mathbf{C}_i$.

Reformulation

Following similar steps as in the single stream case, the proposed beamforming design is transformed to a convex optimization problem

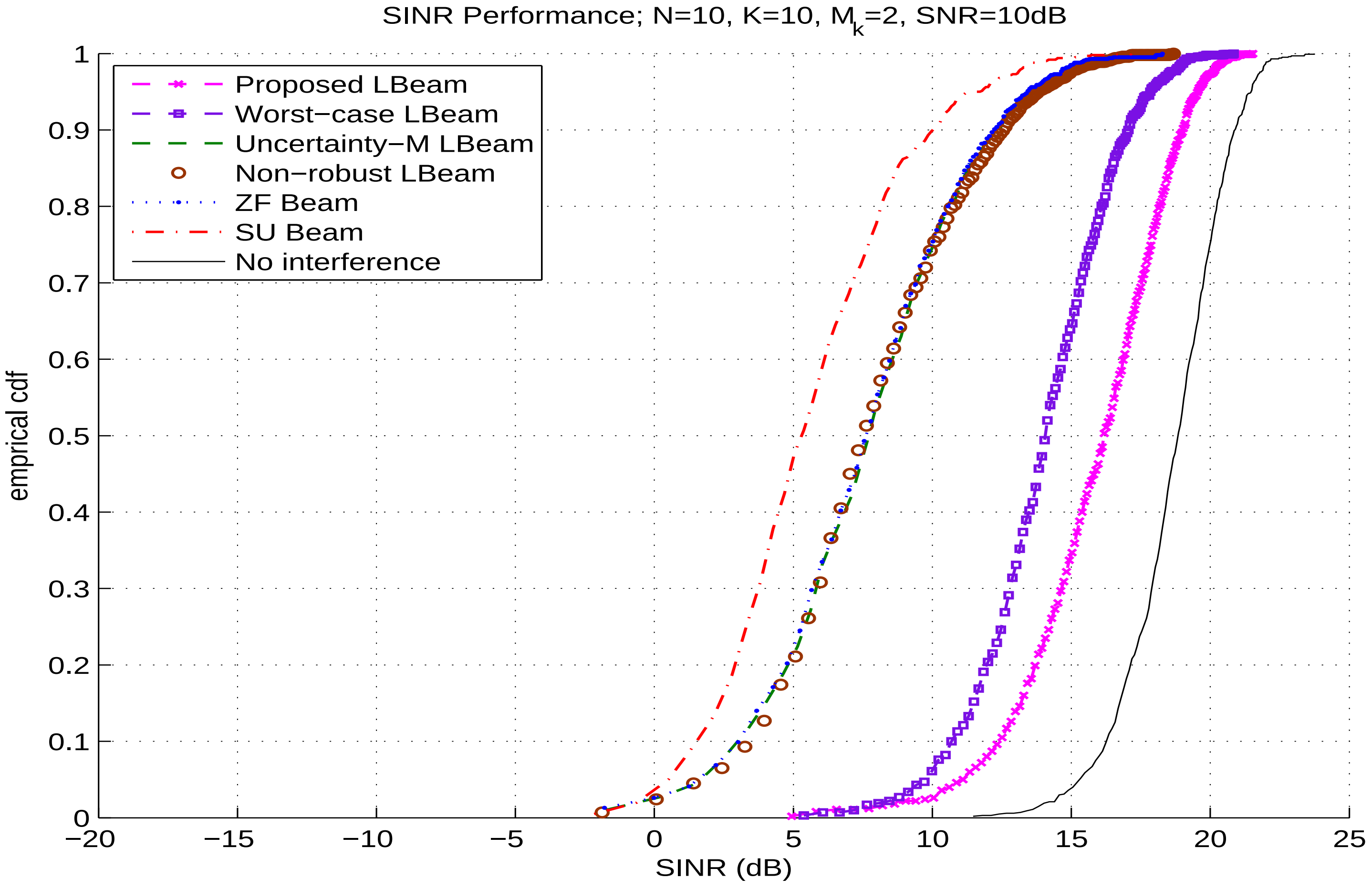
$$\begin{aligned} \max_{\overline{\mathbf{W}}_i} \quad & \mathrm{tr}\left\{(\mathbf{H}_{i_p}^H \mathbf{H}_{i_p} + M_i \sigma_e^2 \mathbf{I}_N)\overline{\mathbf{W}}_i\right\} \ , \\ \text{s.t.} \quad & \mathrm{tr}\left\{\left(\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} + n_i \sigma_e^2 \mathbf{I}_N\right)\overline{\mathbf{W}}_i\right\} \leq p_i \gamma_{th_i} \ , \\ & \mathrm{tr}\{\overline{\mathbf{W}}_i\} \leq 2 \ , \\ & \overline{\mathbf{W}}_i \geq 0 \ , \quad i = 1 \ , \ldots , K \ , \end{aligned}$$

where $\overline{\mathbf{W}}_i = \mathbf{C}_i \mathbf{C}_i^H$ is the design parameter with rank$(\overline{\mathbf{W}}_i) = 2$.

Simulation

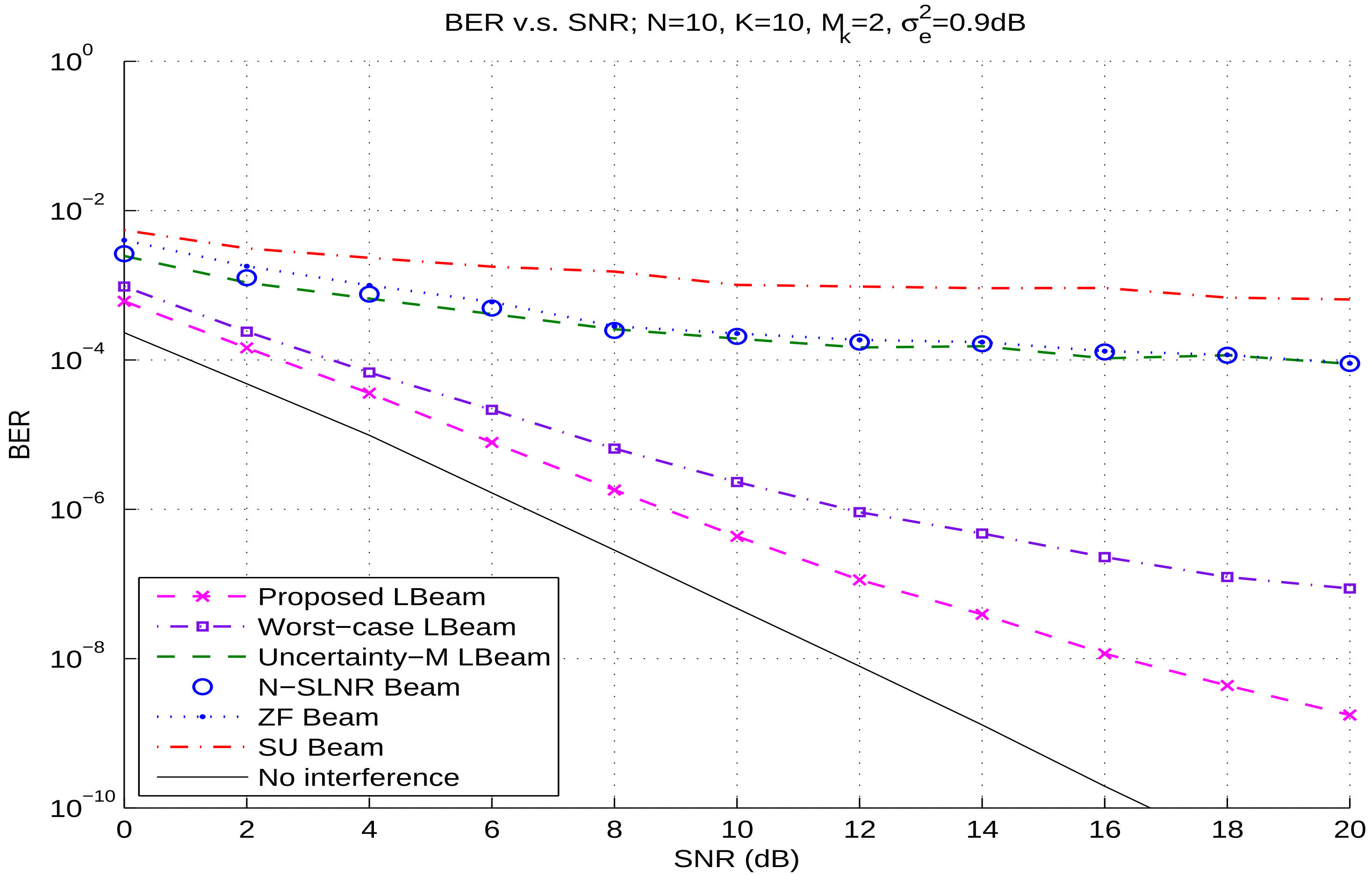
- We consider a single stream MU-MIMO system consisting of $N = 10$ transmit antennas at base station, and $K = 10$ users with $M_k = 2$ receive antennas.
- The reference channels are realizations of i.i.d. complex normally distributed random variables with zero mean and unit variance.
- The variance of AWGN noise per receive antenna is the same for all users, $\sigma_1^2 = \cdots = \sigma_K^2 = \sigma^2$.
- The uncertainty level is set to be $\sigma_e^2 = 0.9$.
- Parameters in prob. constraint: $\gamma_{th_i} = 0.9$, $p_{out} = 5\%$.

SINR Performance



Empirical cdf of output SINR at $\mathrm{SNR} = 10\mathrm{dB}$

BER vs. SNR


BER v.s. SNR; N=10, K=10, M_k=2, σ_e^2=0.9dB
BER
SNR (dB)
10^0
10^-2
10^-4
10^-6
10^-8
10^-10
0
2
4
6
8
10
12
14
16
18
20
Proposed LBeam
Worst−case LBeam
Uncertainty−M LBeam
N−SLNR Beam
ZF Beam
SU Beam
No interference

Conclusions

- We applied the probabilistic constraint approach for robust transmit beamforming design in MU-MIMO systems.
- The proposed method maximizes the average SNR and ensures low leakage power with high probability.
- The probabilistic constraint was transformed to a convex one. Computational complexity is the same as most robust designs.
- The proposed beamformer achieves the highest robustness and best system performance among existing robust designs.

References

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